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Marie Pomarede, Aziz Hamdouni, Erwan Liberge, Elisabeth Longatte, Jean-François Sigrist

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SIMULATION OF FLUID FLOW USING REDUCED-ORDER MODELING BY POD APPROACH APPLIED TO ACADEMIC CASES

Marie POMAREDE

Service Technique et Scientifique DCNS Propulsion 44620 LA MONTAGNE, France

Aziz HAMDOUNI

Laboratoire d'Étude des Phénomènes de Transfert et de l'Instantanéité : Agro-industrie et Bâtiment Université de La Rochelle - Avenue Michel CREPEAU 17042 LA ROCHELLE Cedex 1, France

Elisabeth LONGATTE

LaMSID – UMR CNRS/EDF n²832 1 avenue du Général de Gaulle 92141 CLAMART Cedex, France

ABSTRACT

Tube bundles in steam boilers of nuclear power plants and nuclear on-board stokehold are known to be exposed to high levels of vibrations. This coupled fluid-structure problem is very complex to numerically set up, because of its three-dimensional characteristics and because of the large number of degrees of freedom involved. A complete numerical resolution of such a problem is currently not viable, all the more so as a precise understanding of this system behaviour needs a large amount of data, obtained by very expensive calculations. We propose here to apply the now classical reduced order method called Proper Orthogonal Decomposition to this case. This choice could lead to reduced calculation times and allow parametrical investigations thanks to a low data quantity. But, it implies several challenges inherent to the fluid-structure characteristic of the problem. Previous works on the dynamic analysis of steam generator tube bundles already provided interesting results in the case of non flowing fluid - i.e. quiescent fluid [J.F. Sigrist, D. Broc; Dynamic Analysis of a Steam Generator Tube Bundle with Fluid-Structure Interaction; Pressure Vessel and Piping, July 27-31, 2008, Chicago]. A first step on the implementation of POD in academic cases (one-dimensional equations, 2D-single tube configuration) is presented. Future work will consist in working on the tube bundle configuration, first in the fixed case and then with structure motion allowed. Present study shows the efficiency of the reduced model to reproduce similar problems from a unique data set for various configurations as well as the efficiency of the reduction for simple cases

INTRODUCTION

The running rate of a nuclear power plant or on-board stokehold steam boiler intrinsically induces several vibratory levels, especially concerning the boiler tube bundle [5], [10], [30], [36]. It is shown that fluid-elastic instabilities can occur in such a realistic configuration [8], [11], [12], leading to a certain destruction of one or more tubes: this is why the study

Erwan LIBERGE

Laboratoire d'Étude des Phénomènes de Transfert et de l'Instantanéité : Agro-industrie et Bâtiment Université de La Rochelle - Avenue Michel CREPEAU 17042 LA ROCHELLE Cedex 1, France

Jean-François SIGRIST

Service Technique et Scientifique DCNS Propulsion 44620 LA MONTAGNE, France jean-francois.sigrist@dcnsgroup.com

and a precise comprehension of this vibrations phenomenon is crucial. But, this good comprehension stays difficult because of the high number of parameters that play a role in the generation of vibrations [31], [32]. Thus, the only contribution of experiments and/or on-site statements is not sufficient and it becomes necessary to develop accurate and robust CFD numerical codes [21], [39] in order to set up parametric studies that could help the understanding of violent phenomena like fluid-elastic instabilities.

Another constraint is the high resource level that is necessary to set up this fluid/structure interaction problem: to be as close as possible to real conditions, a fully 3D turbulent flow has to be computed [17], added to the cost of the structure coupling. In an industrial configuration, such a computation is not possible, first because of the resource cost, second because of the CPU time involved.

We propose here an alternative that could offer perspectives in the study of tube bundle vibrations, using reduced models. Theses models are well-known and widely used in the domain of fluid mechanics [9], [15] as well as structure mechanics [2], but they still represent a challenge within the frame of fluid/structure interaction [13], [25], [41]. They however could give a better comprehension of the physics of this fluid/structure interaction problem, making possible the access to some parameters or information. The reduced model that we propose to set up in this paper is the Proper Orthogonal Decomposition (POD) [20], [23], which is now used in many domains [1], [20], [26].

This paper is organized as follows: a first part is dedicated to the main vibrations problems that can encounter a tube bundle in real conditions. Then, actual numerical models that are used to solve and study such a problem are presented in the second part. Proper Orthogonal Decomposition will be described as well as its potential contribution specifically for this crucial question of tube bundle vibrations. Finally, in the third part, first numerical results in the use of POD are proposed and perspectives for a future work are exposed.

1. THE HEAT EXCHANGER TUBE BUNDLE AND ITS VIBRATIONS PROBLEMS

Figure 1 shows the general functioning of an on-board stokehold steam boiler of a water pressurized reactor (WPR). This functioning is quite the same for a civil nuclear steam boiler, which is also a WPR. Water of the primary circuit feeds the tubes (in red) driven by a pump. Liquid water of the secondary circuit comes from the top of the tube bundle and vaporizes by ascension along the tubes.



The tube bundle is constituted of long, fine and numerous tubes that are close from each other. Thus, particularly because of the turbulent flow of the secondary circuit, theses tubes are leaded up to vibrate. A very large number of specific parameters have to be considered while studying this configuration.

Main variables used in this paper are depicted in table 1:

Variable	Definition	
ρ	Fluid density (kg m $^{-3}$)	
μ	Fluid dynamic viscosity (kg.m ⁻¹ .s ⁻¹)	
D	Diameter of one tube (m)	
Р	Step between two tube diameters (m)	
[M]	Total mass matrix	
[C]	Damping matrix	
[K]	Stiffness matrix	
Tab. 1. Main variables of the system		

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Classically, the motion equation of one tube is the following:

$$[M]\{\ddot{Q}(t)\} + [C]\{\dot{Q}(t)\} + [K]\{Q(t)\} = \{F_{ext}\}$$
(1)

where $\{Q(t)\}$ represents the motion generalized coordinates vector, $\{F_{ext}\}$ is the fluid forces vector to which the tube is subjected.

A precise definition of these parameters is of high importance. If we denote D' the diameter of the neighbouring tubes of one tube by considering their confinement, total mass matrix [M] contains the tube mass, the mass of the fluid conveying the tube, and the added mass that is function of the rate D'/D and of the fluid density ρ . The confinement varies according to the tube arrangement. In the same manner, damping can be divided in three energy dissipation mechanisms: friction damping, viscous damping and squeezefilm damping. Experimental data compiled by Pettigrew and Taylor [32], [33] helped them to define semi-empirical formulations for different damping components, according to the thickness of the grid supporting the tubes, the proper frequency of the considered mode, the total mass, and considerations on tube supports. Moreover, these formulations are different considering a single liquid phase, a single gaseous phase or a two phase flow.

Before presenting different tubes excitation phenomena, it is necessary to redefine dimensionless numbers that govern the fluid flow, the Reynolds number and the Strouhal number. Table 2 gathers these dimensionless numbers.

Variable	Description
R _e	$rac{ ho U_p D}{\mu}$
\mathbf{S}_{t}	$rac{f_s D}{U_p}$

Tab. 2. Dimensionless numbers for a fluid-structure interaction problem in a tube bundle configuration

 U_p is the step fluid velocity, it takes into account the tube

confinement. It is defined as: $U_p = U_{\infty} \frac{P}{P-D}$ where U_{∞} is the equivalent mean flow velocity that would have been

imposed in an infinite domain. Four vibratory excitation mechanisms are susceptible to exist under transversal flow: turbulent excitation, vortexinduced vibration, acoustic resonance and fluid-elastic instability.

The apparition of each phenomenon depends on parameters that are not always known or easily observable. But researchers have collected information in order to predict and prevent this apparition in a functioning regime.

Turbulent excitation is unavoidable: Reynolds number of the flow regime is $R_e \in [10^4; 10^7]$. Moreover, turbulence is recommended in order to produce a mixing as perfect as possible, to obtain good heat transfers. But, turbulence induces generally low structural motion amplitude [4] so this mechanism has to be considered in fretting-wear damage considerations [33].

Vortex-Induced vibrations are well-known in the case of a single tube in semi-infinite domain [10], [42]. In the present case of tube bundles, this phenomenon is more complex. The presence of a "lock-in" [19], [22] has been detected by Pettigrew and Gorman [30] in air, but Axisa [4] never encountered the set up of such a mechanism. Pettigrew and Taylor [32] explain that the presence of turbulence tends to reduce the possibility of vortices to set up, that reduces vortex-induced vibrations. Furthermore, Païdoussis [29] insists on the fact that the distinction between both mechanisms (turbulence and vortex shedding) is far from easy.

Acoustic resonance is susceptible to appear in the context of a single gaseous phase exchanger. It strongly depends on the tubes arrangement. This phenomenon is not taken into account within the framework of this study since exchangers are fluid-fluid exchangers. Works on this mechanism have been led in different configurations [8], [43].

Finally, fluid-elastic instability is the most spectacular vibratory excitation phenomenon [7], [35]: it leads to a very quick ruin of the tubes that have been excited. For theses grounds, researchers particularly focused on this mechanism in order to avoid it at all costs. When flow velocity reaches a certain critical threshold V_{c} , structural motion induces a fluid force with the same orientation than the structure motion direction, which leads to vibratory amplitudes much larger than those that are usually observed. Only the tube breaking, caused by repeated impacts between the tube and its support, will stop the excitation. This is precisely an interaction between fluid and elastic efforts, the first feeds the second and conversely: on that point, this mechanism differs from vortexinduced vibrations, which are auto-limited in amplitude. A very large number of models, empirical or semi-empirical, have been set up in the hope of avoiding such a situation. A

very widespread model is Connors' [14], who proposed to express the critical mean fluid velocity V_c as:

$$V_C = K \sqrt{A_R} \tag{2}$$

 V_c is a function of the Scruton number

$$A_R = \frac{2\pi\zeta m}{\rho D^2}$$
 where ζ is the global damping for the

considered mode and m is the lineic total mass. This dimensionless number measures the energy proportion that the system can dissipate thanks to its proper damping, compared to the energy proportion that the fluid provides to the structure through the fluid-elastic coupling force. A constant K weights

this number; it is defined as $K = \sqrt{\frac{\pi}{k_1 k_2}}$ where k_1 and k_2 are

stiffness constants of two neighbouring tubes. When the global damping becomes negative, the system starts to be instable. This model has been enriched by many authors [18], [29], [31]. The notion of a delay between structure solicitations and the reaction of the flow is also introduced. This delay has a big influence on the velocity stability threshold. Price [35] shows that mathematically, a fluid-elastic instability phenomenon is set up by a negative work of fluid efforts. Price highlights three mechanisms that can explain this energy extraction to the fluid by the structure: first, the discrepancy between structure displacement and fluid forces, which presupposes that the damping governs the phenomenon, since it is the physics which is related to structure displacement. When this damping becomes negative, instability appears. The second mechanism appears when at least two degrees of freedom are involved and that there is a phase discrepancy between them. The structure displacement is impacted, that is why the mechanism is described as leaded by stiffness. The third mechanism is the apparition of hysteresis in fluid forces evolution because on non-linearities. Here, efforts amplitude depends on the structure motion direction.

But, flow passing a tube bundle is a system containing a very high number of degrees of freedom, so a precise analytical description of exciting efforts is not possible; moreover, several modes can be excited, considering relatives cylinders motions.

For each vibration mechanism, experimental data have been collected and exploited by various authors in order to define criteria to respect [11], [24], [33]. Sometimes, semianalytical models have been developed, particularly in the case of the fluid-elastic phenomenon; see [35] for example.

CFD models have been set up in order to avoid experimental costs and to observe a large number of parameters. Vortex-induced vibrations have been numerically studied with high precision and most of their mechanisms are now well understood. However, when turbulence is present in the flow, fluid-structure interactions are more difficult to represent, notably because of the three dimensional characteristic of turbulence. Fluid-elastic instability is also very hard to model for the same reason and because of the number of parameters that play a role in its set up.

A constant challenge in numerical modelling is based on interactions between fluid and structure motions. In order to solve these interactions, two classes of approaches exist [27]: the first is called monolithic approach, and consist in the use of a unique formulation for fluid and structure modelling. This approach is theoretically optimal, but costly and only adapted to simple geometries. The second is a partitioned approach: fluid and structure equations are resolved separately, with information communication between both of them. A good description of these approaches can be found in [38].

However, in both cases, a complete numerical resolution of the fluid-structure interaction in a tube bundle in functioning regime cannot be carried out. In this context, the use of Reduced Order Models (ROM) can be a solution to achieve the realization of such a study. A ROM allows solving a problem which formulation contains the bulk of the system information with a reduced number of degrees of freedom.

In the framework of fluid dynamics, the criterion that ensures the fact that "the bulk of the system information" is kept can be an energetic criterion. Using this criterion, the optimal approach is the well-known Proper Orthogonal Decomposition (POD). This method is the subject of next section, as well as its potential applications for the study of fluid-structure interactions in tube bundles.

2. PROPER ORTHOGONAL DECOMPOSITION (POD)

In classic computational fluid dynamics studies, approximated Navier-Stokes equations are computed on a bidimensional of three-dimensional domain Ω for a time interval [0;T]. In the case of a large three-dimensional domain, and if the flow is turbulent, calculation times can be very long. Moreover, if a parametric study has to be set up, it is necessary to lead as many calculations as there are values of the parameter in question to test. The Proper Orthogonal allows saving Decomposition calculation time on computations, and provides a projection basis that can be reused in parametric studies: in an industrial context, theses advantages have to be taken into account.

Proper Orthogonal Decomposition has notably been introduced by Lumley [28] within the framework of coherent structure extraction of turbulent flows. A rigorous description of POD can be found in [20] for example; a large amount of domains are interested in using POD techniques, what leads to variant methods, see [13] or [37]. Here we briefly present the POD formulation.

Let us consider a domain Ω of the set of all real numbers and a time interval [0;T] where T is a real maximal date. Spatial and time variables are respectively $x \in \Omega$ and $t \in [0;T]$. Let v(x,t) be the unknown field, for example the velocity field (unknown of Navier-Stokes equations), with $v(x,t) \in H(\Omega,T)$, *H* is a Hilbert space. Proper Orthogonal Decomposition consist in determining a determinist basis $\{\Phi_n\}_{n=1, N}$ of functions which give the optimum representation of the field v(x,t). *N* is the size of the POD basis.

A practical approach of POD has been proposed by Sirovich [40], it is called Snapshot POD: this method is based on making the most of samples of experimental or numerical data. Let consider M snapshots of the velocity field v(x,t)(these snapshots can be equally taken from an experimental or numerical set), these snapshot have been sampled during a period T. Snapshot POD consist in solving the following eigenvalue problem:

$$\sum_{k=1}^{M} \frac{1}{M} \left(v(t_i), v(t_k) \right)_{L^2(\Omega)} A_k = \lambda A_i$$
(3)

For each i = 1,...,M, where λ contains eigenvalues. Each element of the POD basis is a linear combination of snapshots, coefficients are A_n^k , n = 1,...,N:

$$\Phi_n(x) = \sum_{n=1}^{M} A_n^k v(x, t_n) \qquad n = 1, ..., N$$
(4)

 $\{\Phi_n\}_{n=1,\dots,N}$ The POD basis has interesting characteristics: it is orthonormal and if we study an incompressible flow, each element of the basis (each POD mode) satisfies the incompressibility condition as well as the boundary conditions of the problem. For а given $n \in [1, 2, ..., N]$, the energetic contribution of the POD mode Φ_n is captured by the corresponding eigenvalue λ_n and the eigenvalues are ranked in descending order $(\lambda_1 > \lambda_2 > \dots > \lambda_N)$. Thus, Proper Orthogonal the Decomposition is optimal in an energetic sense.

As the Proper Orthogonal Basis is fully spatial and based on time snapshots, its use within a fluid-structure interaction resolution is not immediate. Indeed, if the numerical sample from which the snapshots are extracted has been obtained thanks to a moving mesh technique, the construction of the POD basis has no sense, since the POD modes are not timedependants. Thus, in the case of fluid-structure interaction problems, an extension of the Snapshot POD is necessary. This has been proposed by Liberge [25] who propose to work on a static spatial domain using a projection of snapshots. Here, we present some POD properties working on very simple cases, as well as first applications on the case we are interested in. When POD modes $\{\Phi_n\}_{n=1, N}$ are determined, a low order dynamical system is solved. For that, the partial differential equations are projected on the POD basis constructed for the field v(x,t). Then, a system of ordinary differential equations, which size N^* is less or equal to the POD basis size, is obtained. To determine this size N^* , an energetic criterion is used, based on eigenvalues of the problem; then the POD basis is truncated to N^* modes.

For example, in the very simple case of the one-dimensional heat transfer equation, written as:

$$\begin{cases} \frac{\partial v(x,t)}{\partial t} - \frac{\partial^2 v(x,t)}{\partial x^2} = 0 & t \in [0,T], x \in \Omega \\ + BC & (5) \\ + IC & \end{cases}$$

Where BC and IC respectively signify boundary conditions and initial conditions; the dynamical system is:

$$\left(\frac{\partial v(x,t)}{\partial t}, \Phi_i\right) + \left(\frac{\partial v(x,t)}{\partial x}, \frac{\partial \Phi_i}{\partial x}\right) = 0$$
(6)

with Φ_i the ith POD mode and if we assume homogeneous boundary conditions. The field v(x,t) projected on the POD basis $\{\Phi_n\}_{n=1,\dots,N^*}$ is the following:

$$v(x,t) = \sum_{n=1}^{N^*} a_n(t) \Phi_n(x)$$
(7)

Thus the low order dynamical system becomes:

$$\frac{da_i}{dt} = -\sum_{n=1}^{N^*} a_n \left(\frac{d\Phi_n}{dx}, \frac{d\Phi_i}{dx} \right)$$
(8)

because of the orthonormal characteristic of the POD modes.

A very interesting characteristic of the POD basis is its ability to represent a solution different from the problem which the basis is computed. Of course, the new problem has to be similar to the first one, which is precisely the case in the framework of a parametric study. An example on the onedimensional heat transfer equation with two different boundary conditions is proposed, based on works of Chinesta [13]: we define a first field with a heat flux step function for boundary condition, and a second field with a heat flux ramp function for boundary condition.

A POD basis is computed from the first problem and both dynamical systems are computed by projection on this unique basis. The reconstruction gives good results, see figure 2.



Fig. 2. POD reconstruction of two similar problems by projection on a unique POD basis

In order to be close to a tube bundle problem, a POD basis is computed for the problem of a single circular cylinder in cross-flow at $R_e = 100$ (the problem of lock-in for such a configuration have been previously studied, see [34]; here the cylinder stays fixed, calculations have been run thanks to Code_Saturne [3]). Figure 3 shows first and second velocity components of the flow at a date t in the sample period [0;T]. Then, figure 4 shows first component of the two firsts POD modes obtained from this sample. The sample period [0;T] corresponds to one period of lift force fluctuations (i.e. around 6 s.) with a time step of $\Delta t = 0.025$ s. 150 snapshots have been taken to constitute the sample. In the case of a low Reynolds number flow around a circular cylinder, just one or two flow periods are necessary to make a good sample, with 100 - 150 snapshots per cycle. In the case of a turbulent flow, more pseudo-cycles are needed to take into account most of the energy of the flow.

If we remember that the first mode energetic contribution is preponderant, it is easy to understand that the first POD mode is linked to the mean flow, while next modes (from Φ_2 to Φ_{N^*}) contain the energy of mean flow fluctuations. As the vortex shedding phenomenon comes from these fluctuations, they are visible only from the second mode.



Fig. 3. Streamwise and cross-stream velocity components at time t for a fixed cylinder in cross-flow at $R_e = 100$



Fig. 4. First component of the two firsts POD modes computed from the instantaneous velocity field for a fixed cylinder in cross-flow at $R_e = 100$

Working with the fluctuant flow velocity means that only fluctuations of the mean flow are taken into account: in that case, the first POD mode corresponds to the second POD mode of the basis computed from the instantaneous field. Figure 5 shows the residual error between the original velocity field and its direct reconstruction by POD. From only 7 modes, the error is less than 1%, which is coherent with results of Liberge [25] for a similar case.



Fig. 5. Residual error (in L^2 norm) between instantaneous velocity field and its POD reconstruction for a fixed cylinder in cross-flow at $R_e = 100$

In the case of higher Reynolds numbers, more POD modes are needed to obtain a good representation of the solution. It implies larger calculation times, especially concerning the computation of dynamical system coefficients: this problem still remains a challenge.

3. FUTURE WORK ON TUBE BUNDLE CONFIGURATION

In future work, the configuration of tube bundle will be study with the POD technique, first in a fixed configuration then allowing one tube motion. An entire tube bundle is not possible to model because of the huge size it would take. That is why only a pattern representing of a tube confined will be used, see figure 5.



Fig. 5. Mesh of the pattern used for the POD study of a tube bundle configuration

As said earlier, the POD study of a fluid-structure interaction is a challenge because of the total spatial characteristic of the POD modes. A classic ALE method [16] to compute the field from which POD modes will be computed would not be correct. That is why techniques with non-moving mesh have to be implemented, see [25].Such methods will be set up in future work.

CONCLUSION

In this paper, the crucial problematic of vibratory excitation of a heat exchanger tube bundle is presented. Fluidelastic instability is one of the most violent vibration mechanisms and a lot of studies have been led in order to define the critical fluid velocity and avoid such a phenomenon. This problematic is well known but not well understood. A way to improve our comprehension of tube bundle vibrations is to work with reduced order models (ROM). The most widespread ROM method, called Proper Orthogonal Decomposition (POD) and its properties are briefly presented. Future work consists in the application of this method to the case of tube bundle.

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